Macmillan Academy

Year 11 into 12
Mathematics Project

Trigonometric Functions and their Applications

You will have heard of sin, cos and tan. You'll remember they can be used to find lengths of sides and sizes of angles in triangles.
Tri – Three (tricycle)
gon – angle (as in polygon)
metry – from metric or measuring

These functions have applications in engineering, music, architecture, weather, tides and in many more fields.

However, their meanings extend further than just triangles. You are going to look at these meanings and some of their applications.
Even though there isn't a triangle with an angle of 490 degrees, sin(490) does have a meaning. Your calculator knows this.
But there is an angle *between 0 and 90* which has the same value.
(It is a multiple of 10...)
Write your answer in the box.

sin (490) = sin(☐)

Use your calculator and circle the odd one out in each row:

sin40	sin140	sin300	sin400	sin500
cos30	cos330	cos390	cos490	cos690
tan10	tan100	tan190	tan370	tan550

There are even NEGATIVE angles.
Fill in these boxes.
All the answers are between -360 and 360 (and end in zero..)
If you remember the graphs of sin, cos and tan, that might help, or you could try google?

sin(-350) = sin(☐) = sin(10) = sin(☐)

cos(☐) = cos(☐) = cos(20) = cos(☐)

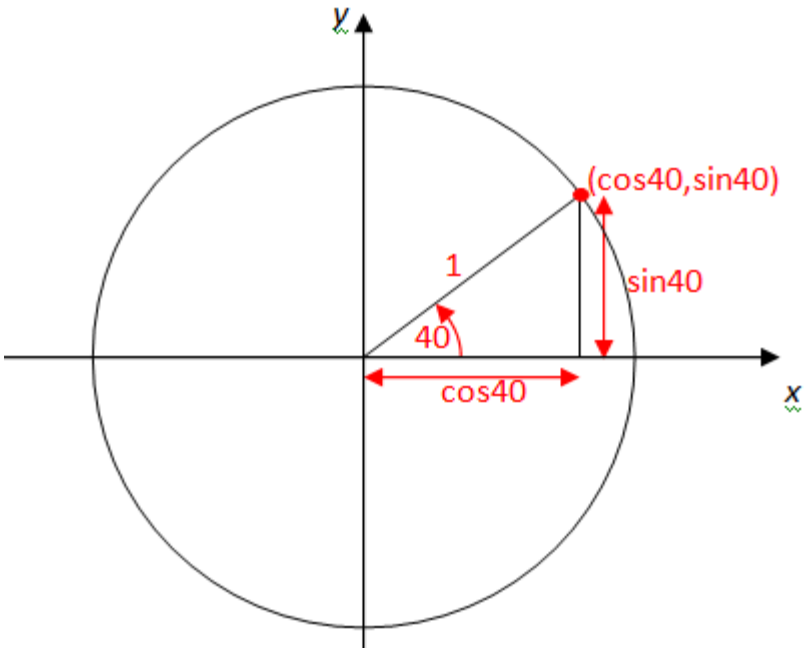
tan(☐) = tan(☐) = tan(30) = tan(☐)

But what is the meaning of the sin/cos/tan of angles bigger than 180 or less than zero?

The fundamental definitions of the trigonometric functions come from the UNIT CIRCLE. (Radius 1, centre at the origin)

The sine (sin for short) of an angle is the y co-ordinate, when the angle is measured anti-clockwise from the x-axis. The cosine (cos for short) is the x co-ordinate.
(When someone decided to have the clocks with the hands starting at the top, going in the direction they do, they got it wrong on both counts.)

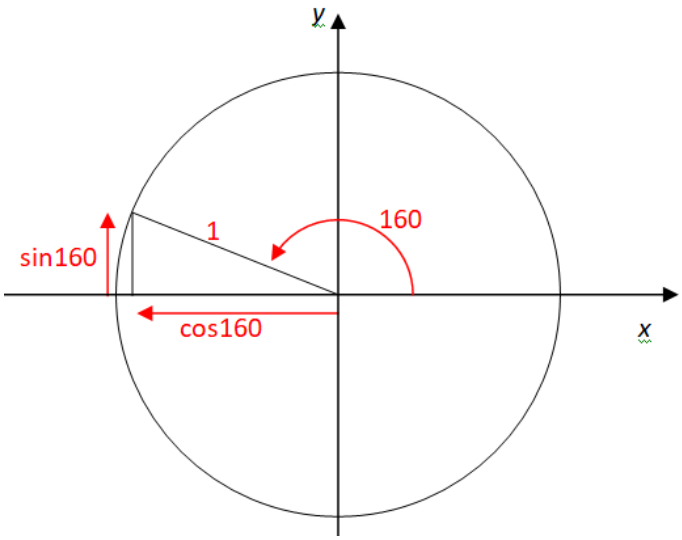
Here is a diagram for an angle of 40 degrees to explain:



Imagine you're doing a GCSE question, to work out the vertical side of the right angled triangle. You would do 1 x sin(40)
Can you see :

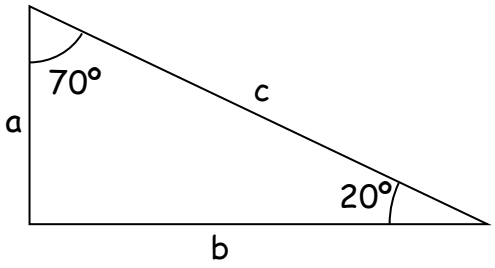
- for angles between 90 and 270 the cosine is negative?
- for angles between 180 and 270 both are negative?
- sin(20) = sin (160) = sin(380)?
- cos(40) = cos(-40)
  (A positive angle goes CW, so a negative one goes...?)

Here is the diagram for 160

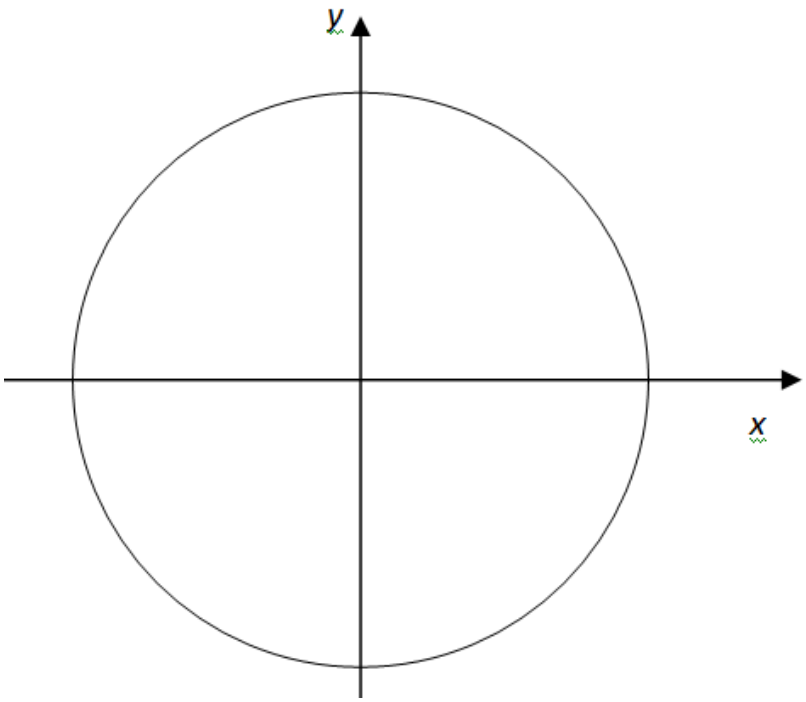


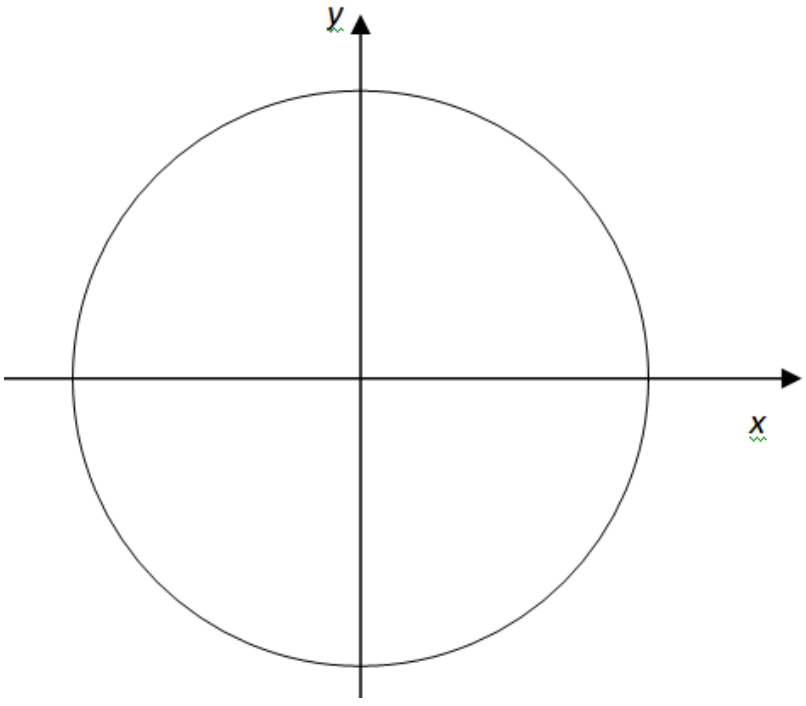
sin(160) is positive, but cos(160) is negative.

You might want to research where the term "sine" comes from.
The "co" in cosine comes from the word complement. Complementary angles add up to 90. You should be able to see from this right angled triangle that sin(20) = cos(70) = $\frac{a}{c}$ and sin(70) = cos(20) = $\frac{b}{c}$



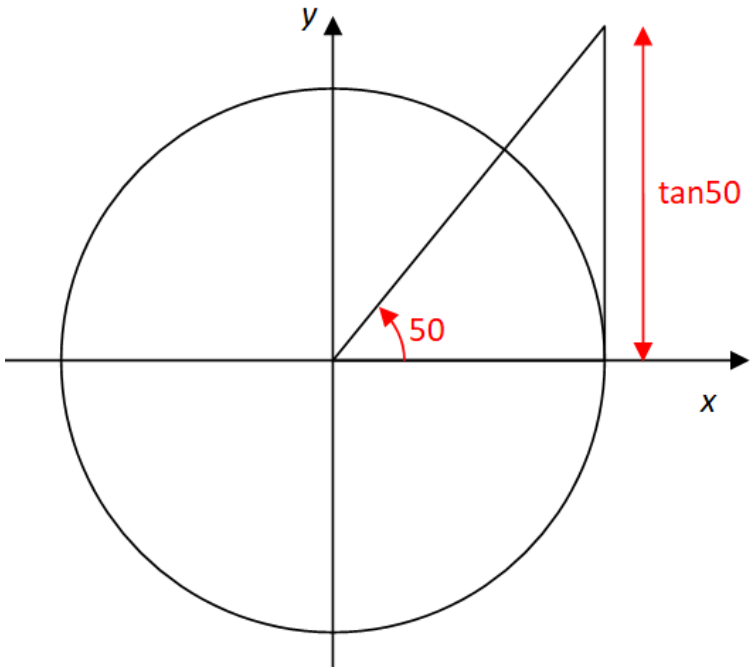
Draw diagrams for angles of 200 and -20





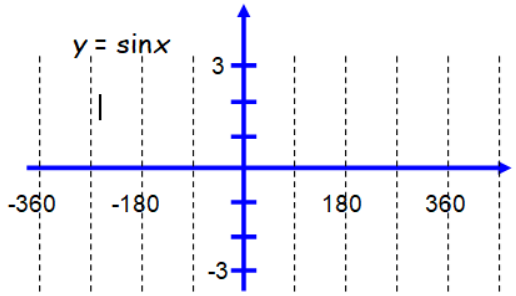
Work out the sin and cos of 200 and -20.
How do they relate to the sin and cos of 20?

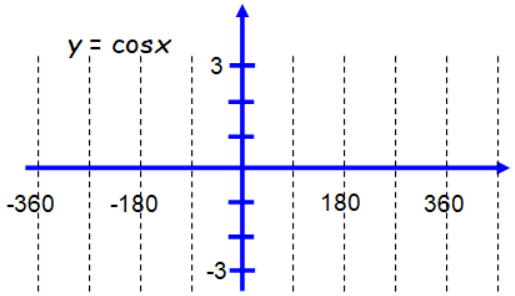
What is tan? You know it as opp/adj, but it best defined using the unit circle. Tan is short for "tangent". Can you see why it is called that?



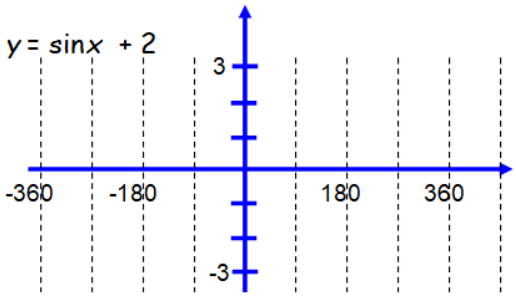
Also tan50 = sin50/cos50. You can check this on your calculator.
You might be able to see why this is from the diagram.

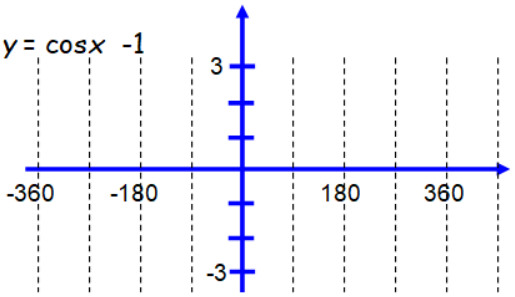
Now draw the graphs of sin, cos and tan. Use google if you've forgotten. Or you could use your calculator.

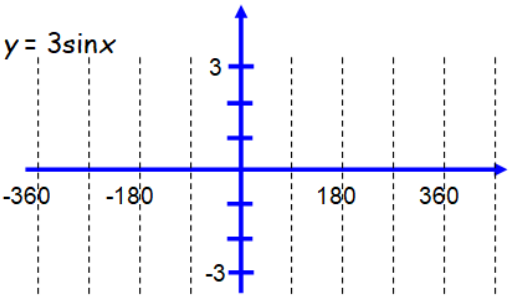


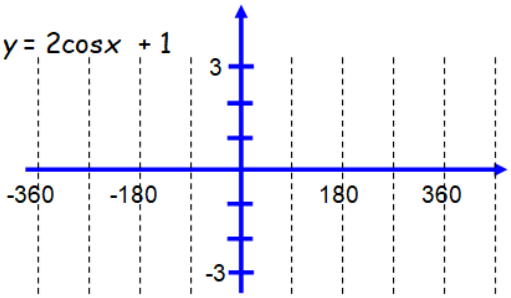


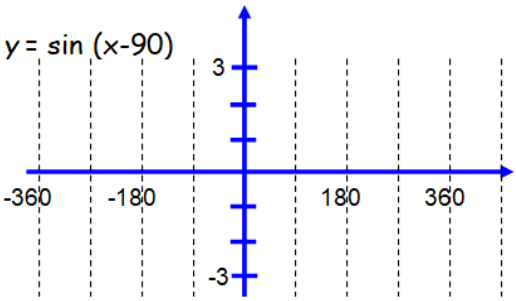
You also need to be able to draw transformations. Use your calculator if you get stuck.

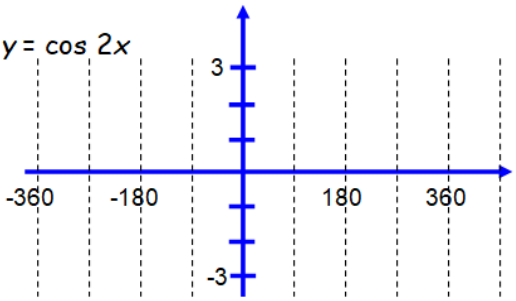












Real life examples.

This is a graph of the length of a day at a particular place in the northern hemisphere.

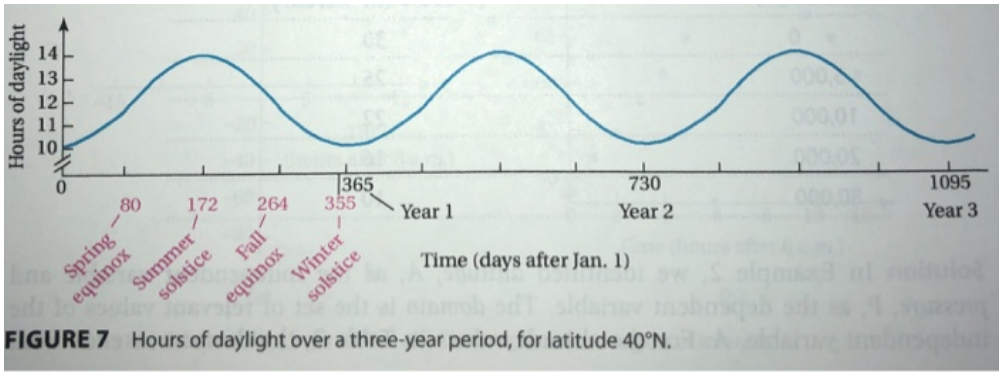
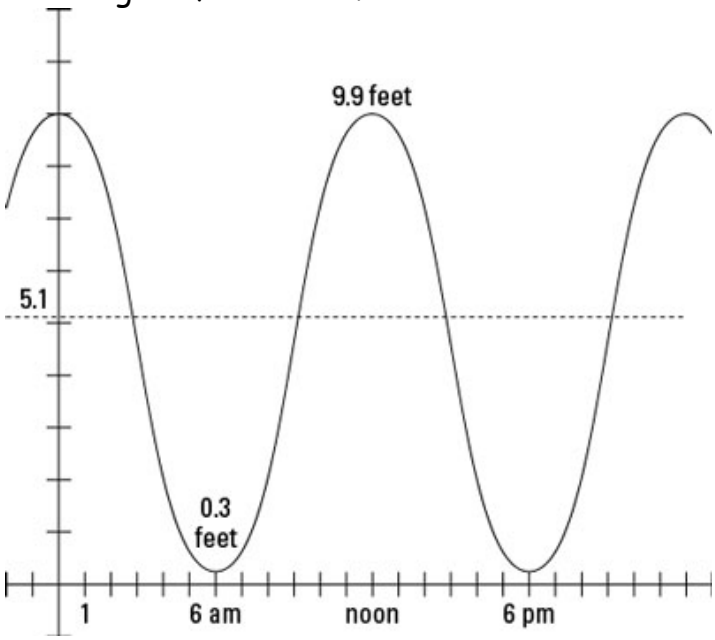
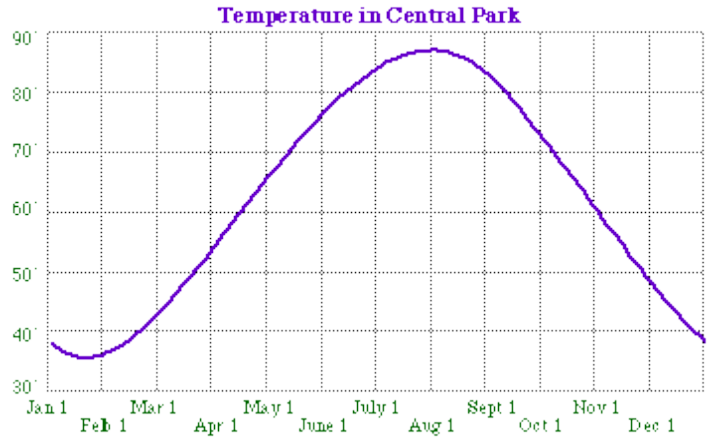


FIGURE 7 Hours of daylight over a three-year period, for latitude 40°N.

This graph shows the height of seawater.



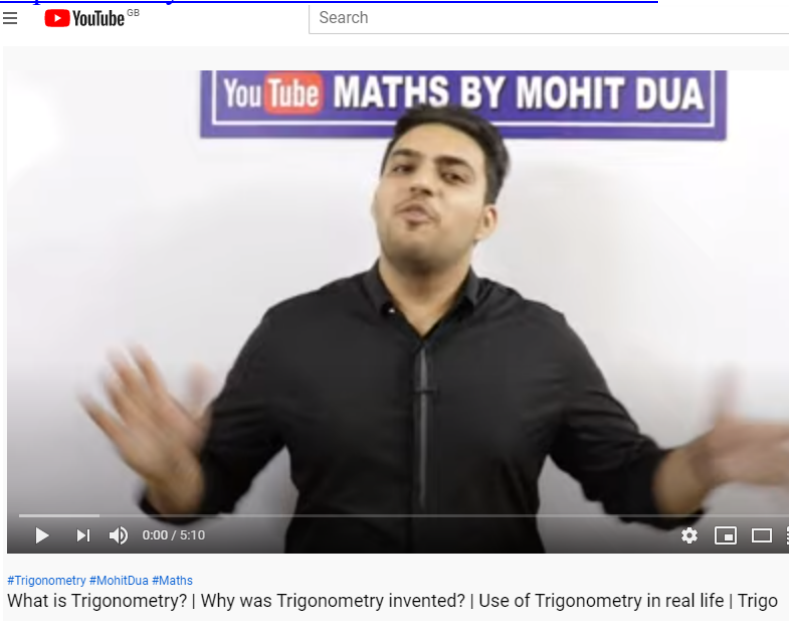
Temperatures in New York (bit warm for the Boro one...)



Search :
Why is trigonometry useful? and watch this video:
https://www.youtube.com/watch?v=WehHFJki9yQ

Also, this is fascinating.
Search:

What is Trigonometry? | Why was Trigonometry invented? | Use of Trigonometry in real life.
https://www.youtube.com/watch?v=bGoWaIIr46M



I'd like you all to really understand the periodic nature of sin, cos and tan.
Circle the ones that are right. Try and use the unit circle definition or the graphs and check with your calculator. Or use the calculator and then refer back to the circle and graphs to understand why.

sin10 =	sin170	sin70	sin370	sin(-190)	sin530
cos30 =	cos(-30)	cos330	cos390	cos490	cos690
tan20 =	tan(-160)	tan200	tan390	tan560	tan740

Now try and find all the solutions to this equation between -360 and 360.
You will need to use the $\sin^{-1}$ function on your calculator and either the graph of sinx, or the unit circle.

$$sinx = 0.695$$

(There are four solutions, Write your answers to the nearest whole number)

Now find all the solutions to these equations between -360 and 360.

$$cosx = 0.809$$

$$tanx = 0.364$$

Can you suggest method to find all the solutions? Your calculator gives you the first one, but how do you get the other four.

Now try these equations

Solve each equation for x in the interval $0 \leq x \leq 360$.

Give your answers to 1 decimal place where appropriate.

a $\sin(x-60)^\circ = 0.5$	**b** $\tan(x+30)^\circ = 1$	**c** $\cos(x-45)^\circ = 0.2$
d $\tan(x+30)^\circ = 0.78$	**e** $\cos(x+45)^\circ = -0.5$	**f** $\sin(x-60)^\circ = -0.89$
g $\cos(x+45)^\circ = 0.9$	**h** $\sin(x+30)^\circ = 0.14$	**i** $\cos(x-60)^\circ = 0.6$

For a, just find the possible answers for x-60 and add 60 to the two answers. You should get:

$$x = 90, 210$$

Here are some of the answers, one for each question, jumbled up and with an extra one. There are two solutions for each question

$x = 142.0,$

$x = , 326.5$

$x = 302.9,$

$x = 113.1$

$x = 8.0,$

$x = 340.8$

$x = 15,$

$x = 227.5$

$x = 75,$

Which is the extra one?

Now try these:

Solve each equation for x in the interval $0 \leq x \leq 180°$.

Give your answers to 1 decimal place where appropriate.

a $\tan(2x + 30°) = 1$ **b** $\sin(2x - 15°) = 0$ **c** $\cos(2x + 70°) = 0.5$

d $\sin(2x + 210°) = 0.26$ **e** $\cos(2x - 38°) = -0.64$ **f** $\tan(2x - 56°) = -0.32$

Here is the solution for a

a $2x + 30 = 45, 180 + 45$

$= 45, 225$

$2x = 15, 195$

$x = 7.5°, 97.5°$

You can check your solutions by subbing your answers back in the original equation.

Eg for a

tan(2x2.5 + 30) = tan(45) = 1

tan(2x 97.5 + 30) = tan(225) = 1